Double line to ground

Boundary Condition

$I_{fa} = 0$

$(I_{fb} + I_{fc}) Z_f = V_{bk} = V_{ck}$

$$I_{fa}^{(0)} + I_{fa}^{(1)} + I_{fa}^{(2)} = 0$$

$$\begin{pmatrix} V_{ka}^{(0)} \\ V_{ka}^{(1)} \\ V_{ka}^{(2)} \end{pmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{pmatrix} V_{ka} \\ V_{kb} \\ V_{kb} \end{pmatrix}$$

$$V_{ka}^{(1)} = \frac{1}{3} (V_{ka} + aV_{kb} + a^2 V_{kb})$$

$$V_{ka}^{(2)} = \frac{1}{3} (V_{ka} + a^2 V_{kb} + a V_{kb})$$

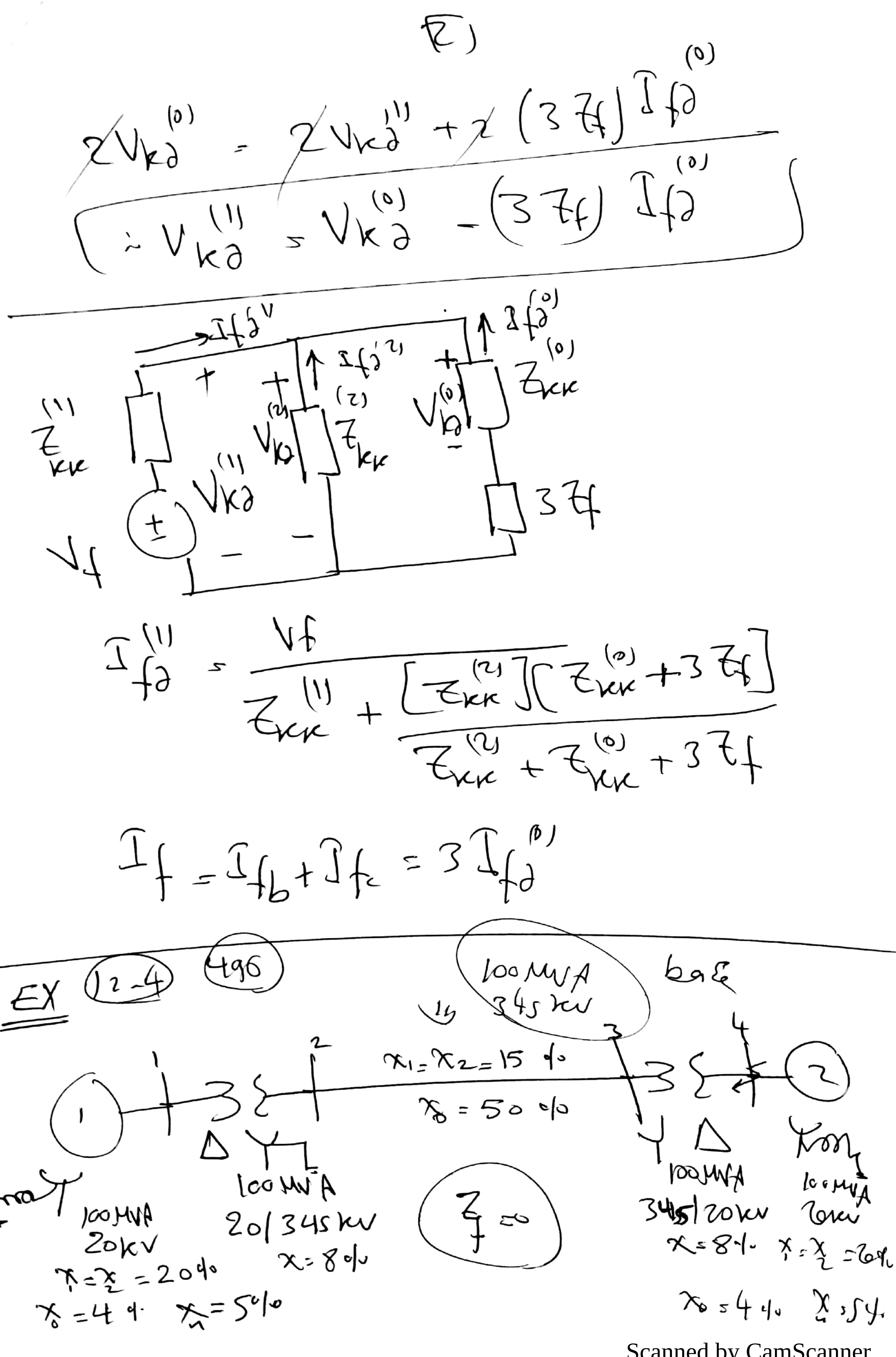
$$\therefore V_{ka}^{(1)} = V_{ka}^{(2)}$$

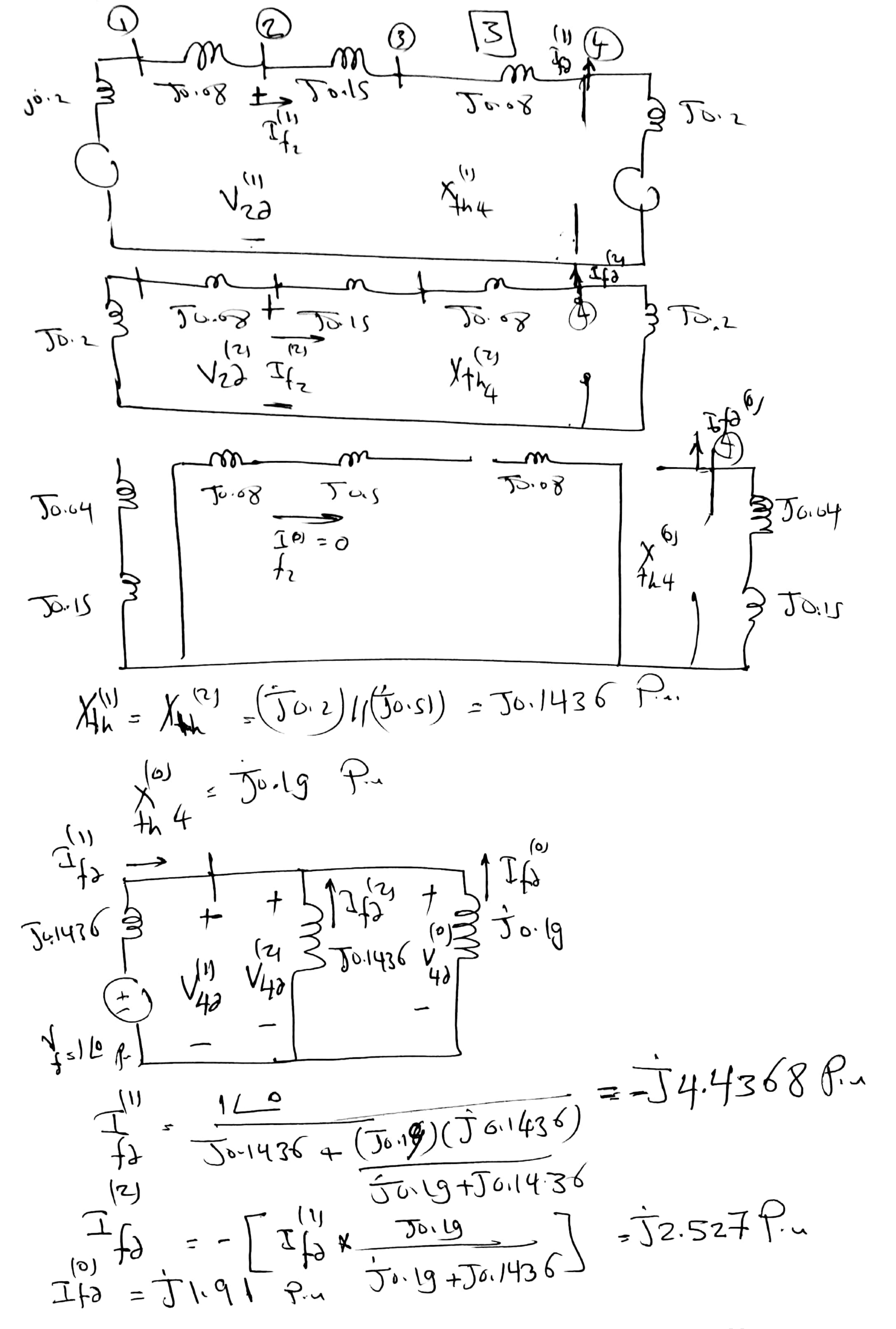
$$V_{ka}^{(0)} = \frac{1}{3} [V_{ka} + V_{kb} + V_{kb}]$$

$(I_{fa} + I_{fb} + I_{fc}) Z_f = V_{kb}$ (zero: I_{fa})

$$3 I_{fa}^{(0)} Z_f = V_{kb}$$

$$V_{ka}^{(0)} = \frac{1}{3} \left(V_{ka}^{(0)} + V_{ka}^{(1)} + V_{ka}^{(2)} + 2(3 I_{fa}^{(0)} Z_f) \right)$$


2)
2Vkd(0) = 2Vkd(1) + 2(3Zf) Ifd(0)
∴ Vkd(1) = Vkd(0) − (3Zf) Ifd(0)
Ifd(1)
Ifd(2)
Ifd(0)
Zkk(1)
Zkk(2)
Zkk(0)
Vkd(1)
Vkd(2)
Vkd(0)
3Zf
Vf
Ifd(1) = Vf / (Zkk(1) + [Zkk(2)][Zkk(0) + 3Zf] / (Zkk(2) + Zkk(0) + 3Zf))
If = Ifb + Ifc = 3Ifd(0)
EX 12-4 496
100 MVA 345 kV base
x1 = x2 = 15 %
x0 = 50 %
100 MVA 20 kV
x1 = x2 = 20%
x0 = 4 % xn = 5%
100 MVA 20/345 kV x = 8 %
Zf = 0
100 MVA 345/20 kV x = 8%
x0 = 4 %
100 MVA 20 kV x1 = x2 = 20%
x0 = 4% xn = 5%


3
1
2
3
4
J0.2
J0.08
J0.15
J0.08
J0.2
I_f2^(1)
I_fa^(1)
V_2a^(1)
X_th4^(1)
I_fa^(2)
J0.2
J0.08
J0.15
J0.08
J0.2
V_2a^(2)
I_f2^(2)
X_th4^(2)
I_fa^(0)
J0.64
J0.08
J0.15
J0.08
J0.64
I_f2^(0) = 0
X_th4^(0)
J0.15
J0.15
X_th^(1) = X_th^(2) = (J0.2)||(J0.51) = J0.1436 P.u
X_th4^(0) = J0.19 P.u
I_fa^(1)
I_fa^(0)
J0.1436
I_fa^(2)
J0.19
V_4a^(1)
V_4a^(2)
J0.1436
V_4a^(0)
V_f = 1∠0 P.u
I_fa^(1) = 1∠0 / (J0.1436 + (J0.19)(J0.1436)/(J0.19+J0.1436)) = -J4.4368 P.u
I_fa^(2) = -[I_fa^(1) * J0.19/(J0.19+J0.1436)] = J2.527 P.u
I_fa^(0) = J1.91 P.u

$$I_f = 3 I_{fa}^{(0)} = 3 * j1.91 = j5.729 \text{ P.u}$$

$$I_b = \frac{100 * 10^6}{\sqrt{3} * 20 * 10^3} = 2886.75 \text{ A}$$

$$\boxed{I_f = j16.54 \text{ KA}}$$

$$V_{4a}^{(0)} = V_{4a}^{(1)} = V_{4a}^{(2)} = -j1.91 * j0.19$$

$$= 0.3629 \text{ P.u}$$

$$\begin{pmatrix} V_{4a} \\ V_{4b} \\ V_{4c} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \begin{pmatrix} 0.3629 \\ 0.3629 \\ 0.3629 \end{pmatrix}$$

$$= \begin{pmatrix} 1.0887 \\ 0 \\ 0 \end{pmatrix}$$

$$V_{4ab} = 1.0887 * \frac{20}{\sqrt{3}} \text{ KV}$$

$$V_{4bc} = 0$$

$$V_{4ca} = -1.0887 * \frac{20}{\sqrt{3}} \text{ KV}$$

EX 12.5 (499)

$$I_{f_2}^{(1)} = I_{f_2}^{(1)} * \frac{j0.20}{j0.71} = -j4.4368 * \frac{j0.2}{j0.71} = -j1.2498$$

$$I_{f_2}^{(2)} = j2.527 * \frac{j0.2}{j0.71} = j0.7118 \text{ p.u}$$

$$V_{2a}^{(1)} = 1\angle 0 - \left[~~-j4.4368~~ - j1.2498 * (j0.2 + j0.08) \right]$$

$$= 0.65 \text{ p.u.}$$

$$V_{2a}^{(2)} = -\left[I_{f_2}^{(2)} * (j0.2 + j0.08) \right] = 0.1993 \text{ p.u.}$$

$$V_{2A}^{(1)} = 0.65 \angle 30$$

$$V_{2A}^{(2)} = 0.1993 \angle -30$$

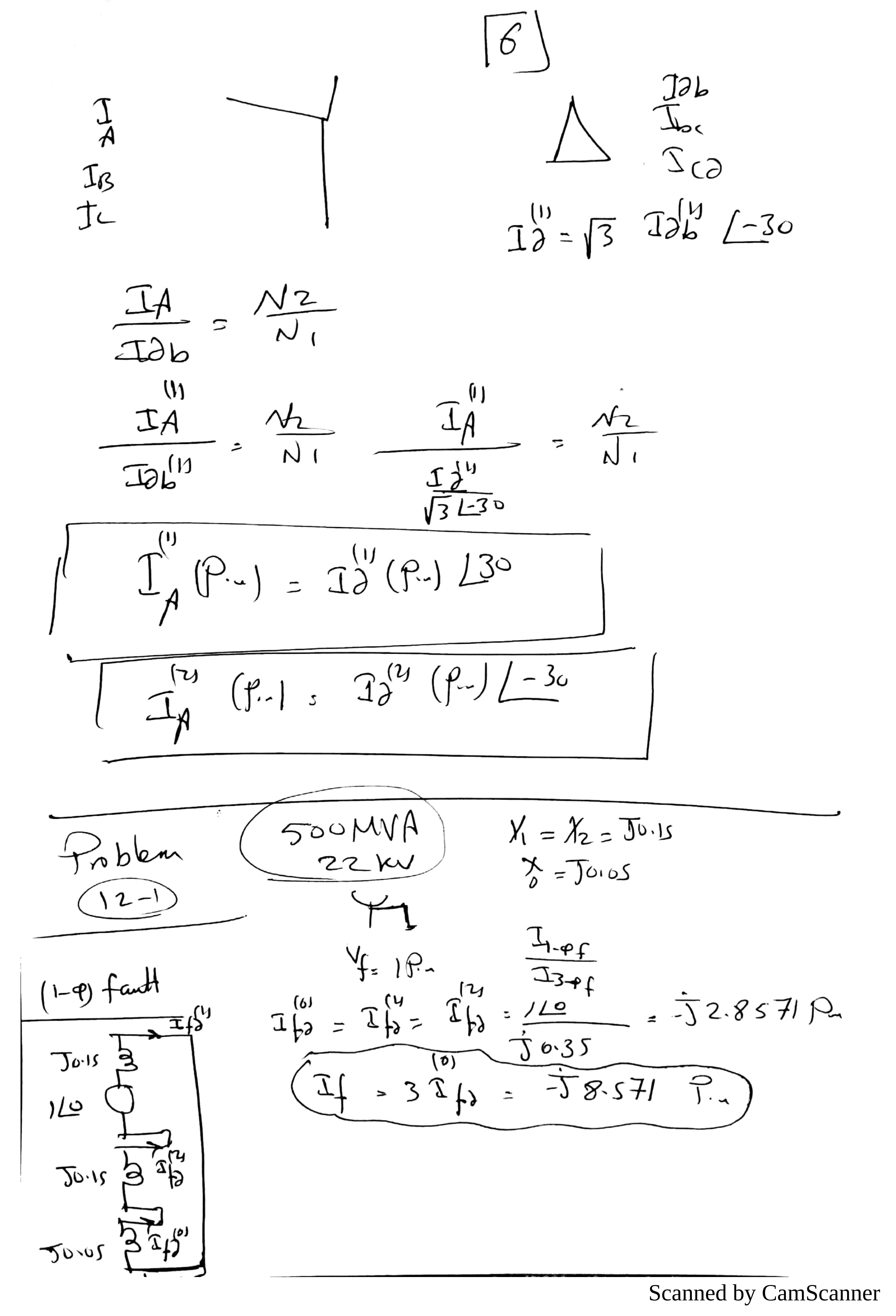
$$\begin{bmatrix} V_{2A} \\ V_{2B} \\ V_{2C} \end{bmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \begin{pmatrix} 0 \\ 0.65 \angle 30 \\ 0.1993 \angle -30 \end{pmatrix}$$

$$= \begin{pmatrix} 0.7693 \angle 17 \\ 0.5798 \angle 107.3 \\ 0.5798 \angle 167.3 \end{pmatrix}$$

$$V_{2A} = 0.7693 \angle 17 * \frac{345}{\sqrt{3}} \text{ kV}$$

$$V_{2B} = 0.5798 \angle 107.3 * \frac{345}{\sqrt{3}} \text{ kV}$$

$$V_{2C} = 0.5798 \angle 167.3 * \frac{345}{\sqrt{3}} \text{ kV}$$


6
IA
IB
IC
Iab
Ibc
Ica
Ia(1) = √3 Iab(1) ∠-30
IA / Iab = N2 / N1
IA(1) / Iab(1) = N2 / N1
IA(1) / (Ia(1) / √3 ∠-30) = N2 / N1
IA(1) (P.u) = Ia(1) (P.u) ∠30
IA(2) (P.u) = Ia(2) (P.u) ∠-30
Problem
12-1
500MVA
22kV
X1 = X2 = J0.15
X0 = J0.05
Vf = 1P.u
I1-φf / I3-φf
(1-φ) fault
Ifa(0) = Ifa(1) = Ifa(2) = 1∠0 / J0.35 = -J2.8571 P.u
If = 3 Ifa(0) = -J8.571 P.u
Ifa(1)
J0.15
1∠0
J0.15
Ifa(2)
J0.05
Ifa(0)

(3-φ) fault

$$I_f = \frac{1}{j0.15} = -j6.67 \text{ P.u}$$

j0.15

1∠0

$$\frac{I_{f_{1-\varphi}}}{I_{f_{3-\varphi}}} = \frac{8.571}{6.67} = 1.298$$

12-2 $\frac{I_{llf}}{I_{3-\varphi f}}$

$$I_{fa}^{(1)} = -I_{fa}^{(2)} = \frac{1\angle 0}{j0.3} = -j3.33 \text{ P.u}$$

$$I_{fa}^{(0)} = \text{zero}$$

$I_{fa}^{(1)}$ $I_{fa}^{(2)}$

j0.15 j0.15

1∠0

$$I_f = I_{f_b} = -I_{f_c}$$

$$I_{fb} = \cancel{I_{fa}^{(0)}} + a^2 I_{fa}^{(1)} + a I_{fa}^{(2)}$$

$$= (1\angle 240)(-j3.33) + (1\angle 120)(j3.33)$$

$$= -5.773 \text{ P.u}$$

$$\begin{pmatrix} I_{fa} \\ I_{fb} \\ I_{fc} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} \begin{pmatrix} I_{fa}^{(0)} \\ I_{fa}^{(1)} \\ I_{fa}^{(2)} \end{pmatrix}$$

$$\frac{I_{llf}}{I_{3-\varphi}} = \frac{5.773}{6.67} = 0.867$$

12-3

j0.15

1∠0

j0.15

j0.05

$3X_n$

$$I_f = 3 I_{fa}^{(0)} = 3 \frac{1\angle 0}{j0.15 + j0.15 + j0.05 + 3X_n}$$

$$= -j6.67$$

$$X_n = j0.0326 \text{ P.u}$$

$$Z_b = \frac{(22)^2}{500}$$

$$X_n = 0.0326 \, Z_b = 0.0321 \, \Omega$$

12-4

$$\frac{I_{1-\phi f}}{I_{3-\phi}} = 1.0$$

$$\frac{I_{llf}}{I_{3-\phi f}} = 0.86$$

double to ground fault

$I_{f_a}^{(1)}$, $I_{f_a}^{(0)}$

j0.15, j0.15, j0.05, $3X_n$ j0.1 P.u, $1\angle 0$

$I_f = 3 I_{f_a}^{(0)}$

$$I_{f_a}^{(1)} = \frac{1\angle 0}{j0.15 + j0.075} = -j4.44 \text{ P.u}$$

$I_{f_a}^{(0)} = j2.22$ P.u

$\therefore I_f = 3 \times j2.22 = j6.66$ P.u

$$\frac{I_{f_{dlg}}}{I_{f_{3\phi}}} = \frac{j6.66}{j6.67} \simeq 1.0$$

12-6

100 MVA
20 kV
$X_1 = X_2 = 20\%$
$X_0 = 5\%$
$X_n = 0.32\ \Omega$
$V_f = 1\angle 0$

$$Z_b = \frac{(20)^2}{100} = 4\ \Omega$$

$$X_n = \frac{0.32}{4} = 0.08 \text{ P.u}$$

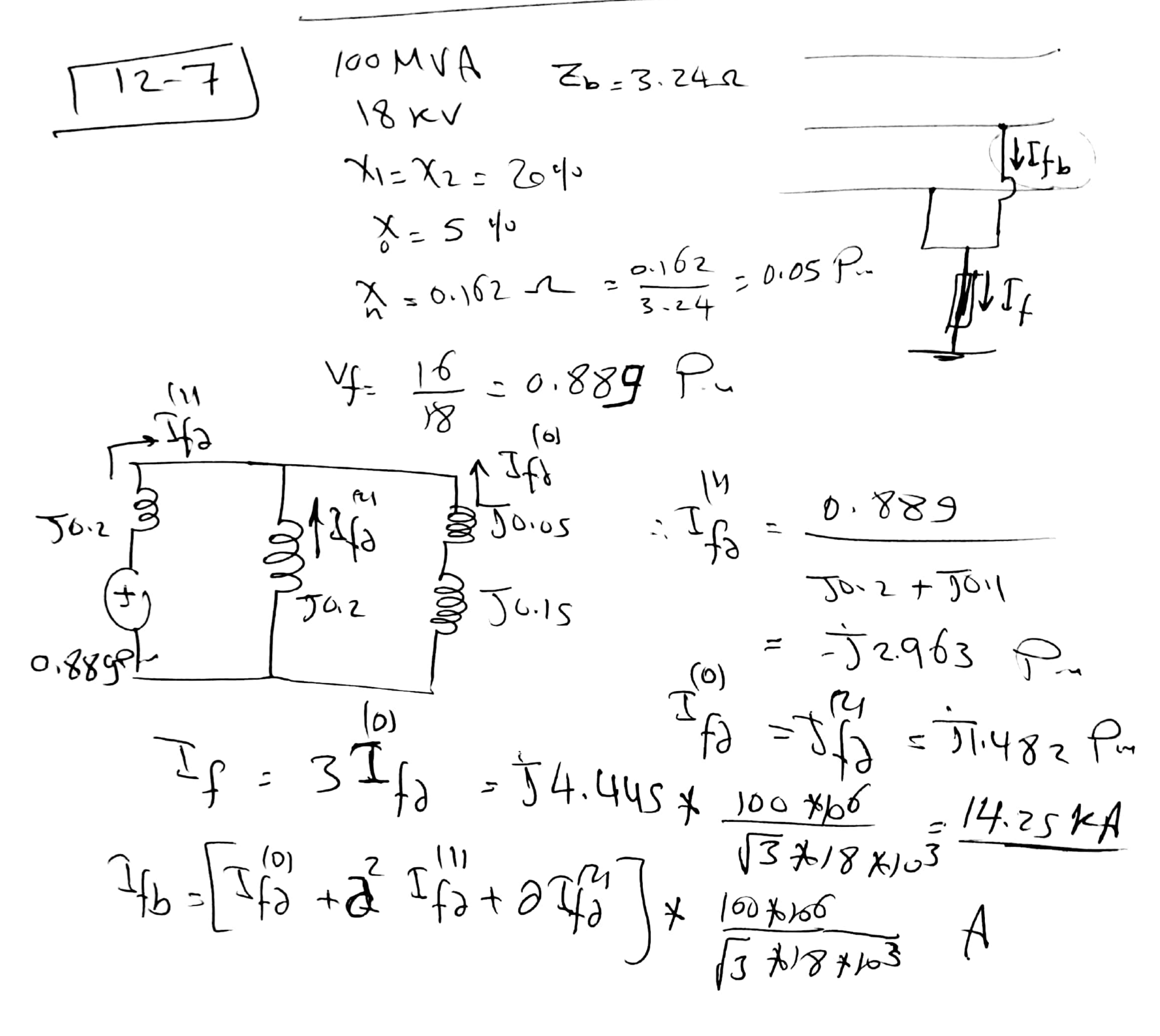
9

$$I_{fa}^{(0)} = I_{fa}^{(1)} = I_{fa}^{(2)} = \frac{1\angle 0}{j0.45 + j0.24}$$

$$= -j1.449 \text{ P.u}$$

$$I_f = 3 I_{fa}^{(0)} = -j4.347 \text{ P.u}$$

$$I_b = \frac{100*10^6}{\sqrt{3}*20*10^3} = 2886.7\text{A}$$



12-7

100 MVA $\quad Z_b = 3.24\Omega$

18 kV

$X_1 = X_2 = 20\%$

$X_0 = 5\%$

$X_n = 0.162\ \Omega = \frac{0.162}{3.24} = 0.05$ Pu

$V_f = \frac{16}{18} = 0.889$ Pu

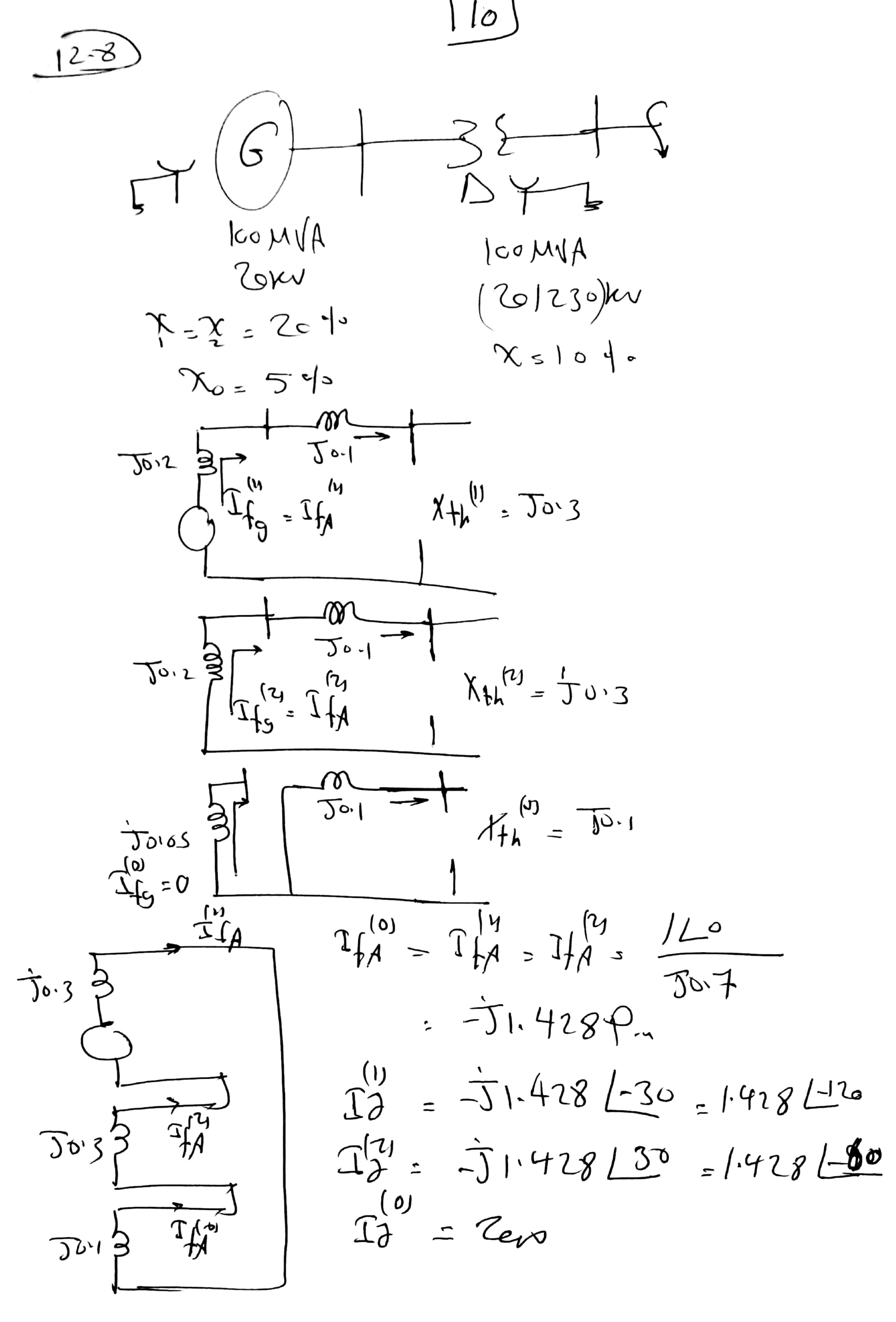
$$\therefore I_{fa}^{(1)} = \frac{0.889}{j0.2 + j0.1}$$

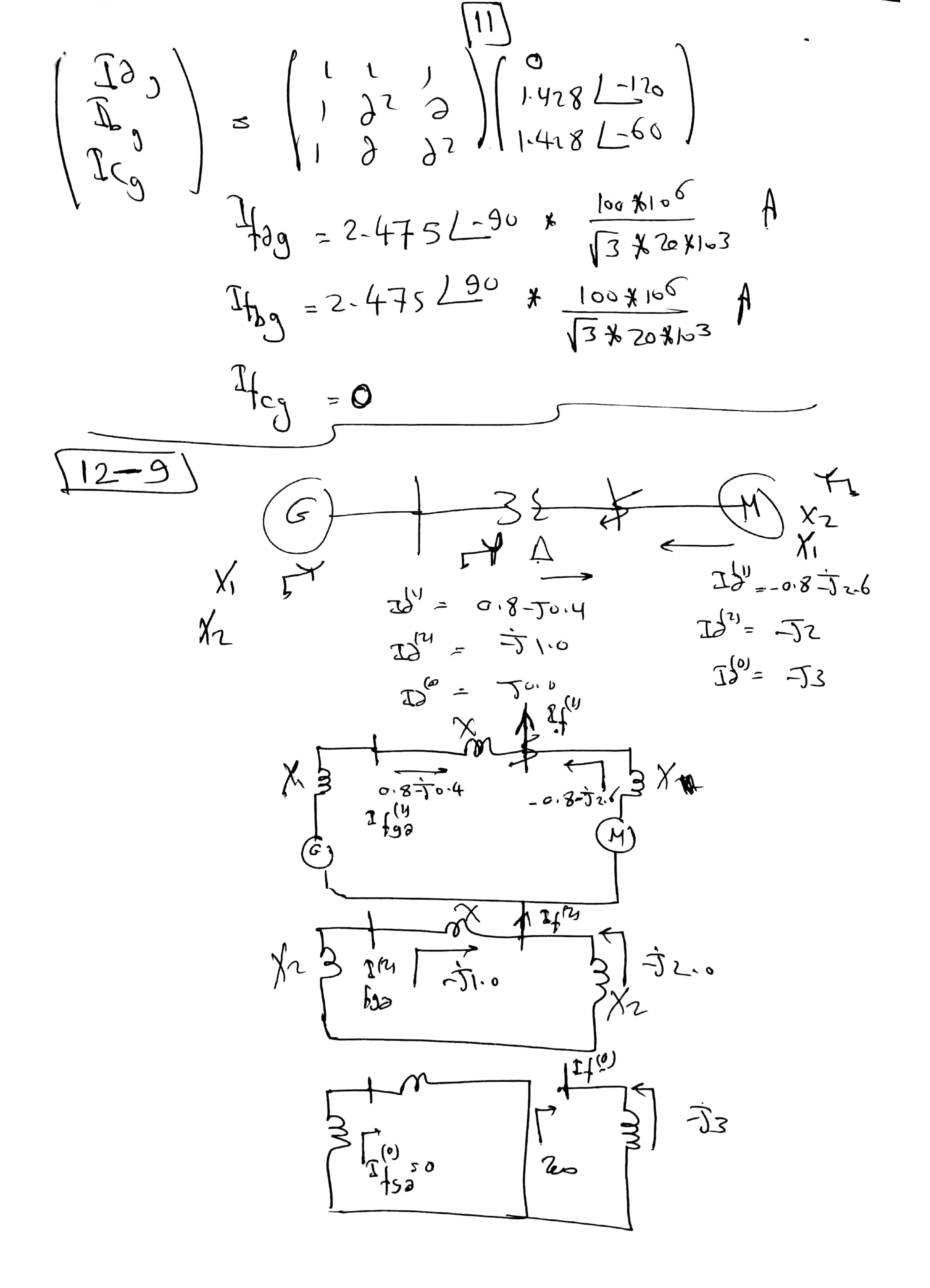
$$= -j2.963 \text{ Pu}$$

$$I_{fa}^{(0)} = I_{fa}^{(2)} = j1.482 \text{ Pu}$$

$$I_f = 3 I_{fa}^{(0)} = j4.445 * \frac{100*10^6}{\sqrt{3}*18*10^3} = \underline{14.25 \text{ kA}}$$

$$I_{fb} = \left[I_{fa}^{(0)} + a^2 I_{fa}^{(1)} + a I_{fa}^{(2)} \right] * \frac{100*10^6}{\sqrt{3}*18*10^3} \text{ A}$$


10
12-8
G
D Y
100 MVA
20 kV
100 MVA
(20/230) kV
X1 = X2 = 20 %
X0 = 5 %
X = 10 %
J0.2
J0.1
If_g(1) = If_A(1)
Xth(1) = J0.3
J0.2
J0.1
If_g(2) = If_A(2)
Xth(2) = J0.3
J0.05
If_g(0) = 0
J0.1
Xth(0) = J0.1
If_A(1)
J0.3
J0.3
If_A(2)
J0.1
If_A(0)
If_A(0) = If_A(1) = If_A(2) = 1∠0 / J0.7
= -J1.428 P.u
Ia(1) = -J1.428 ∠-30 = 1.428 ∠-120
Ia(2) = -J1.428 ∠30 = 1.428 ∠-60
Ia(0) = Zero


11
Iag
Ibg
Icg
1 1 1
1 a2 a
1 a a2
0
1.428 ∠-120
1.428 ∠-60
Ifag = 2.475 ∠-90 * 100*10^6 / √3 * 20*10^3 A
Ifbg = 2.475 ∠90 * 100*10^6 / √3 * 20*10^3 A
Ifcg = 0
12-9
G
M
X1
X2
Δ
Ia(1) = 0.8-j0.4
Ia(2) = -j1.0
Ia(0) = j0.0
Ia(1) = -0.8-j2.6
Ia(2) = -j2
Ia(0) = -j3
If(1)
X
0.8-j0.4
-0.8-j2.6
Ifga(1)
If(2)
Ifga(2)
-j1.0
-j2.0
If(0)
Ifga(0) = 0
zero
-j3

$$I_{fa}^{(1)} = -0.8 - j2.6 + 0.8 - j0.4 = -j3 \text{ P.u}$$

(12)

$$I_{fa}^{(2)} = -j2.0 - j1.0 = -j3 \text{ P.u}$$

$$I_{fa}^{(0)} = -j3 + 0 = -j3 \text{ P.u}$$

$$I_{fa}^{(0)} = I_{fa}^{(1)} = I_{fa}^{(2)} = -j3.0$$

single line to ground fault

$$I_f = 3 I_{fa}^{(0)} = -j9 \text{ P.u}$$

$$I_{fm}^{(1)} = I_f^{(1)} * \frac{j(X + X_1)}{j(X + X_1 + X_2)}$$

$X_1 = X_2$

$$I_{fm}^{(2)} = I_f^{(2)} * \frac{j(X + X_2)}{j(X + X + X_2)}$$

$$\therefore I_{fm}^{(1)} = I_{fm}^{(2)}$$

$\therefore I$ Prefault $I_{fg}^{(1)} - I_{fg}^{(2)} = 0.8 - j0.4 - (-j1.0)$

$= 0.8 + j0.6$ P.u

$$I_{f_{gA}}^{(0)} = 0$$

$$I_{f_{gA}}^{(1)} = (0.8 - j0.4) \angle 30$$

$$I_{fgA}^{(2)} = -j1.0 \angle -30$$

$$\therefore \begin{pmatrix} I_{fgA} \\ I_{fgB} \\ I_{fgC} \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{pmatrix} * \begin{pmatrix} 0 \\ (0.8 - j0.4)\angle 30 \\ -j1.0 \angle -30 \end{pmatrix}$$

$I_A = 0.903 \angle -64.3$ P.u

$I_B = 1 \angle -53.1$ P.u

$I_C = 1.893 \angle 121.6$ P.u

12-15